1. (*16 points*) Consider $f(x)$ as defined below and complete the subsequent tasks.

$$f(x) = \begin{cases} 2\,x - 4 & x < 3 \\ x^2 - x - 4 & x \geq 3 \end{cases}$$

(a) Calculate

$$\lim_{x \to 3^-} f(x)$$

(b) Calculate

$$\lim_{x \to 3^+} f(x)$$

(c) Calculate

$$\lim_{x \to 3} f(x)$$

(d) Decide if $f(x)$ is a continuous function or not. Make sure you support your claim with rigorous reasoning.

2. (*10 points*) Calculate

$$\lim_{x \to 5} \frac{x^2+5\,x-50}{25-x^2}$$

3. (*10 points*) Calculate

$$\lim_{x \to \infty} \frac{13-11\,x^2}{3-5\,x+7\,x^2}$$

4. (*16 points*) Calculate the derivative of $f(x) = x^{e} - e^{-x} - \frac{1}{\ln(x)} + \frac{1}{x^3}$

5. (*18 points*) Calculate the derivative of the following functions. Simplify only if appropriate.

(a) $f(x) = \left(1 - x + x^5\right)\left(1 - x^2 + x^7\right)$

(b) $g(x) = \dfrac{\ln(x)}{1+x^4}$

(c) $h(x) = \sqrt{\ln x} - e^{\sqrt{x}}$

6. (*20 points*) Sketch the graph of $f(x) = \frac{x^4}{2} + \frac{2\,x^3}{3} - x^2 - 2\,x$. List the coordinates of where the **extrema** or **points of inflection** occur. State where the function is **increasing** or **decreasing**, as well as where it is **concave up** or **concave down**. Also decide based on your graph if there is any x-**intercept** other than the origin.

7. (*15 points*) Calculate $\int \left(x^2 - \sqrt{x} + e^x - \frac{1}{\sqrt{x}} + 2 \right) dx$.

8. (*15 points*) Complete the following tasks.

(a) Calculate $\int_0^1 x - \sqrt{x}\ dx$

(b) Calculate $\int_1^e \frac{1}{x}\ dx$

(c) Calculate $\int_0^1 x^2\, e^{2x^3}\ dx$

9. (*20 points*)

(a) Determine the area under the graph of $f(x) = 5 - 4x + 4x^3$ between $x = -1$ and $x = 1$

(b) Calculate the **average** of $f(x)$ on $[-1, 1]$.

10. (*20 points*) An archaeological survey has produced a well-preserved specimen that has been sent to a laboratory for further testing. It has been determined that the amount of radioactive ${}^{14}C$, remaining is 7.0% of the amount present at the time of death. Assume the half-life of ${}^{14}C$ is 5,730 years and let $A(t) = A_0\, e^{k\,t}$ be the amount of ${}^{14}C$ in the specimen after t years.

(a) Is k positive or negative here?

(b) Use the information about the half-life and determine the exact value of k.

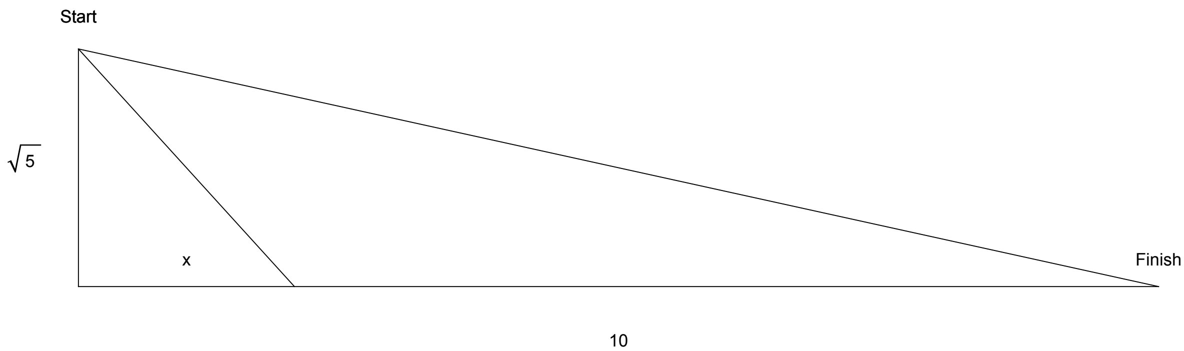
(c) Is it necessary to know A_0 to determine the age?

(d) Determine the exact value of the age of the specimen.

(e) Give a numerical approximation of the age of the specimen rounded to the nearest thousand years. Also indicate why this may exaggerate the accuracy available based on the data of the problem.

11. (*20 points*) Determine the area of the bounded region determined by the curves $y = \sqrt{x} + x^2$. and $y = x + \sqrt{x}$.

12. (*20 points*) A combined swim-and-run adventure race starts at an island that is perpendicularly north of a rugged shore that runs straight W to E. The finish is 10 miles east of the perpendicular nearest point on the shore. Assume a competitor swims at a speed of 2 miles per hour and along the shore is capable of jogging at 3 miles per hour. The island is about $\sqrt{5}$ miles from the shore.



Hint: Recall that the time t it takes to travel a distance d at a speed of v is given by $t = \frac{d}{v}$.

(a) What would the finishing time be if the competitor swims the straight south and then runs the 10 miles along the shore?

(b) What would the time be if the competitor swims all the way to the finish along a straight line? **Hint:** Pythagoras.

(c) Is it possible to improve on the fastest time in **(a)** and **(b)** by 'cutting the corner' and swim along a straight line to some point east of the perpendicular, and then finish by running the remainder along the shore? If so, where should the competitor land and what is the time it takes to complete the race in this case?